THE NUMBER OF MONEYERS AND THE FREQUENCY OF RECOINAGES DURING THE NORMAN PERIOD: A MATHEMATICAL MODEL

I. DAVID BROWN

Introduction

THE Normans, like the late Anglo-Saxons, recoined the circulating silver currency every few years, introducing a new design (type) at each recoinage, a system known as the *renovatio monetae*.[1] Some hundred leading citizens drawn from the major towns across England manufactured coins in the workshops under their control. In the outlying centres, this was presumably primarily intended to produce coin for the payment of taxes, but the major mercantile centres often had several moneyers and produced significant quantities of coin to support trade. Each moneyer was provided with at least one pair of dies, the obverse bearing the name and portrait of the king as the issuing authority, and the reverse bearing name of the moneyer and the town in which he worked. As a result, the late Anglo-Saxon and Norman coinages constitute a remarkable database from which to explore aspects of life in England at the beginning of the second millennium. This resource is all the more important because of the paucity of written records. The names, and where possible the lives, of the moneyers have been extensively examined over the years, but some important questions remain unanswered, such as how many moneyers were in office at any given time, and how often were the recoinages held.[2]

Because the currency supply was frequently recoined, there was little time for coins to be lost, with the result that the information provided by the coins that have survived is incomplete. We may suppose that at any given time, between one and two hundred moneyers were active in some fifty to sixty cities and towns. Allen has recently published a list of all the known Norman moneyers and the types in which they were active.[3] This list is far from a complete record since the calculations presented here suggest that on average about one third of the moneyers active in a given type during the Norman period are unrepresented in the corpus of surviving coins. We do not know if the number of moneyers remained the same from one type to the next, we do not know whether the recoinages occurred on a regular schedule, and evidence for the duration of each of the types is virtually non-existent. As a result, attempts to date each of the different types has necessarily been largely conjectural.[4]

The principal datable events relevant to this coinage are the accession of William the Conqueror (1066), the accession of Henry I (1100), the purge of the moneyers (1124–5), the accession of Stephen (1135), and his death in 1158. The accession of William II in 1087 left no record in the coinage since no distinction was made between the two Williams, unless the PAXS type refers to the proclamation of the King's Peace at William II's coronation.[5] These dates divide the coinage into four epochs: the coins in the name of William, the coins of Henry types 1–14, Henry type 15, and Stephen. This article presents a statistical study of the database of known moneyers for the first two of these epochs, with some comments on the third. The coinage of Stephen is not addressed because of the complications caused by the civil war that occupied most of his reign.

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1 Brown 1997; Allen 2012a, 35–40.
2 Allen 2012a, 12–14, 35–6; Blackburn 1990, 60–2, 65–72.
3 Allen 2012b.
4 Allen 2014, 90–4.
5 Archibald 1984, 324, 328; Allen 2012a, 25–6.

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There are twenty-seven types between 1066 and the purge of the moneyers in 1124: thirteen of these bear the name of William and fourteen the name of Henry. For convenience I have numbered the whole sequence as a single series from 1 to 27, with type 28 (Henry type 15) following the purge of the moneyers. Because of the paucity of coins from the reign of Henry I there has been much discussion about the correct sequence of these types.[6] I have followed the traditional numbering introduced by Brooke, here labelled H1 to H15, but I have employed the ordering used in Allen's list of moneyers, which is almost the same as that of Blackburn but with types H7 and H8 reversed.[7] The Brooke numbers for William I and William II are given as W1–1 to W1–8 and W2–1 to W2–5, for types 1 to 8 and 9 to 13 respectively.

Background

Allen has recently reviewed the evidence for the dating of the different recoinages under William I and William II as well as the related problem of the duration of the different types.[8] The evidence is remarkably thin. There is good documentary evidence for the purge of the moneyers at Christmastide 1124–25, and this is believed to coincide with the transition between types 27 and 28 (H14 and H15), where many moneyers are dropped and mints closed.[9] Although Stewart has presented arguments why H15 might not have been introduced until several years after 1124,[10] type H15 is now widely accepted as having been introduced at the time of the purge. The accession dates of William I and Henry I proved a *terminus post quem* for the coins bearing their names. We can assume that the thirteen types carrying the name of William were issued during the 34 years between 1066 and 1100, for an average of 2.6 years per type, and the first fourteen of the fifteen types carrying the name of Henry spanned the 24 years between 1100 and 1124, for an average of 1.7 years per type. From these averages we can deduce that the duration of each type cannot have been constant over the whole period from 1066 to 1124. It must have changed at least once, though we cannot rule out the possibility that every type had a different duration.

Apart from the accession dates of the kings and the purge of 1124, there is little convincing evidence for assigning dates to individual types. The cluster of hoards from York that end with type 2 (W1–2) can reasonably be associated with the Northern Rebellion of 1068–69 and the subsequent 'harrying of the North' in 1069–70, suggesting that the introduction of type 2 should be dated to no later than 1069, giving a duration of not more than three years for type 1 (W1–1).[11] It has also been argued that type 8 (W1–8), bearing the letters PAXS on the reverse, should be the first type of William II, since variations on the word PAX are found on the first types of both Edward the Confessor and Harold II, but the PAX type of Henry I is his third.[12] The significance of PAX appearing on the coins is perhaps not properly understood. If PAX is indeed the first type of William II, type 8 (W1–8) would have been issued starting in 1087, but such dating, while plausible, is speculative.

Eadmer's *Historia novorum* (written in about 1115) mentions an edict, issued by Henry I around the same time as another edict known to have been issued at Whitsun 1108, requiring that 'no penny or halfpenny should be whole', so that the coins were snicked in the edge before they were issued in order to show that they were of good silver and not plated.[13] This snicking first appears in type 19 (H6) and continues up to type 28 (H15).[14] Type 19 should therefore be dated to around 1108.

Allen has reviewed suggested associations of coin hoards with events of known date, but he notes that while such an association can be made when a cluster of hoards is found in a known

6 Blackburn 1990, 55–62.
7 Blackburn 1990, 55–62; Allen 2012a, 138, 141; Allen 2012b.
8 Allen 2014, 90–4.
9 Blackburn 1990, 64–8.
10 Stewart 1989.
11 Allen 2014, 92.
12 Archibald 1984, 324, 328; Allen 2012a, 25–6.
13 Rule 1884, 193; Blackburn 1990, 62–4.
14 Allen 2009, 98–9.

conflict zone, as in the case of the York hoards mentioned above, it is dangerous to associate a single hoard with a known historical event without additional evidence.[15] For example, it is tempting to associate the enormous Beauworth hoard,[16] buried during type 8 (W1–8), with an historical event, but which one? Dolley suggested that the hoard may have been buried in fear of a Danish invasion in 1085,[17] but Metcalf argued that it might be associated with the distribution of alms from William I's treasury after his death in September 1087.[18] As both are feasible, and there may have been other reasons for the deposition of the hoard, even the Beauworth hoard cannot help us fix the date of type 8. These hints are all we have to date the different types, but any scheme proposing a chronology must respect the limits imposed by the available external evidence.

In addition to this external evidence, some internal clues are provided by the coinage itself. Apart from the drastic changes that took place between types 27 and 28 (H14 and H15), and the snicking of the coins between types 19 and 28 (H6 and H15), which can be associated with external evidence, Blackburn noted that between types 23 and 24 (H11 and H10 respectively) there was a deliberate change in the mint names at least two of the mints (Lincoln and York),[19] and Stewart pointed out that between types 25 and 26 (H12 and H13) there was apparently a deliberate increase in the flan size and possibly the weight of the pennies.[20] Allen has recently argued that there was a reduction in the weight standard during the issue of type 14 (H1) or at the inception of type 15 (H2), an increase in type 22 (H7), a second reduction in type 24 (H10), and two further increases in type 26 (H13) and in type 28 (H15).[21]

It has been suggested that the recoinages occurred at the same time of year, possibly at Michaelmas (29 September) or Lady Day (25 March), because it would make the administration easier, but there is no independent evidence for this.[22] If this were so, we would expect the duration of most types to be integral numbers of years with some accommodation being made at the end of a reign, unless by chance the king's death coincided with the date of a recoinage. If each recoinage occurred at different times of year, durations of non-integral numbers of years would be expected.

The considerable amount of work done tracing the careers of individual moneyers, both through their coins and in the fragments of documentary evidence, has been summarized by Allen.[23] While the results of these studies give a good picture of the careers of some individual moneyers as important citizens of the time, it is difficult to get an overall estimate of the number of moneyers who were active during any one type, except in the few cases where the number of surviving coins is large enough to ensure that coins of virtually every moneyer have survived. The Beauworth hoard ensures that we know that the total number of moneyers who were active during type 8 (PAXS, W1–8)) is likely to be not much more than 178, but for most of the other types it is difficult to extrapolate from the known moneyers to an estimate of the total number, particularly in cases such as the early types of Henry I where the number of surviving coins is much less than the expected number of moneyers.

In his discussion of moneyers and mints in the Anglo-Saxon and Norman periods, Allen summarizes the various estimates of the numbers of moneyers, showing that during the late Anglo-Saxon period the numbers varied widely from type to type and mints were opened and closed often for no apparent reason, suggesting that the authorities were reacting to various pressures, which might range from the demands for more coin to lobbying by influential individuals.[24] The same pressures may well have been responsible for the variable weight of the coins during this period. But the picture changes in the Norman period where, apart from

15 Allen 2014, 92–3.
16 Thompson 1956, 11–13, no. 37.
17 Dolley 1966, 16–17.
18 Metcalf 1988, 13–14.
19 Blackburn 1990, 62
20 Stewart 1989.
21 Allen 2012a, 138–41.
22 Allen 2012a, 35–6.
23 Allen 2009, 2012a.
24 Allen 2012a, 12–23.

Henry's obsession with false coin, one is left with the impression that the kings had better things to do than meddle with the coinage.

Blackburn attempted to estimate the total number of moneyers in the Henry I series by assuming that if a moneyer is known to have been active in types 1 and 3, for example, he was also active in type 2.[25] He used this to determine the correct order of the types, which had been a matter of discussion since Andrew's original attempt nearly a century earlier.[26] He argued that the number of moneyers estimated for a given type using this procedure will be smallest if the different types are arranged in their correct chronological order. On this basis he proposed the ordering that was later adopted (with one minor variation) by Allen in his list of moneyers.[27] Allen's ordering is used in this report. Although Blackburn's approach compensates for some of the missing moneyers, it cannot estimate the number of moneyers whose names are not known at all. Blackburn's estimates of the number of moneyers are relative, and their absolute values are clearly influenced by the number of surviving coins.

The analysis of the number of moneyers

This article describes a more formal statistical approach to extracting information about the numbers of moneyers, one that is independent of the number of surviving coins. Specifically it addresses the question of how many moneyers were active at different times during the Norman period, and incidentally leads to an estimate of the duration of each of the different types between 1066 and 1124. The input to the study is given in Table 1. It includes two types of information: the column labelled m in Table 1 shows the number of moneyers whose names are given by Allen in each of the twenty-eight types between type 1 (W1–1) and type 28 (H15),[28] and the columns labelled b show the number of moneyers known to have been active during a particular type who are also known to have been active in one or more of the subsequent types. The mathematical Appendix shows how this information can be analysed in a way that does not depend on the number of surviving coins, to estimate three important numbers. One is the *total number* of moneyers active at any given time, the second is the fraction of the moneyers active in a given type (t) who were still active in a subsequent type ($t+i+1$). This is known as the *retention rate*. From the retention rate is possible to derive the *duration* of each type, that is the number of years during which it was issued.

If as moneyers retired, died or relinquished their dies for other reasons, we assume that the rate of replacement was roughly constant, we could expect the retention rate from one type to the next to be smaller the longer the duration of the earlier type, unless there were a large decrease in the total number of active moneyers such as occurred in 1124. By comparing the retention rates of the different types one can detect changes in the *type duration*. Taken together with the estimates that indicate periodic changes in the number of active moneyers, such analysis leads to a picture of the mint practices of the time and yields a plausible chronology that conforms to the external evidence. For reasons discussed in the Appendix, several features of the coinage between 1066 and 1124 conspire to make this analysis much more accurate than one would normally expect from a statistical study of this kind.

To understand the process, let us consider the 136 moneyers whose names are known in type 2. We know from Table 1 that at least 68 were still active in type 3, because they are recorded as being among the 96 moneyers known for that type. However, it is probable that several more of those known to us in type 2 were still active in type 3 even though their names are not known for type 3. This is because the 96 known moneyers of type 3 are only a proportion of all the moneyers who were active in the type, and can therefore be expected to include only the same proportion of the moneyers retained from type 2. We cannot compute this directly because we do not know the total number of moneyers active in type 3, nor do we

[25] Blackburn 1990, 60–2, 65–8.
[26] Andrew 1901.
[27] Allen 2012b.
[28] Allen 2012b.

know the retention rate of type 2 moneyers. Fortunately statistical techniques allows us to obtain an expression using the numbers in Table 1 to give the ratio between these two unknown quantities: the total number of moneyers active in type 3 and the retention rate, i.e., the proportion of type 2 moneyers who remained active in type 3, namely:

The total number of moneyers of Type 3, divided by the retention rate of Type 2 (these are the unknown quantities) is equal (subject to statistical uncertainty) to the number of known moneyers of Type 2 multiplied by the number of known moneyers of Type 3 divided by the number of moneyers known in both types, all these latter numbers being known and listed in Table 1. [This is equation A1 in the Appendix.]

In the example above, the total number of moneyers of type 3, divided by the retention rate of type 2, is 136×96/68 = 192, as shown in the row labelled type 3 and column labelled $n_{t,0}$ in Table 2. The retention rate is unknown but it cannot be larger than 1 which would be the case if all the moneyers in type 2 were active in type 3, that is, the number of moneyers active in type 3 cannot be larger than 192. This number, 192, is of course not an exact number since it is subject to statistical uncertainty, and an initial estimate of the size of this uncertainty is indicated by the number (35) in parentheses below 192. It indicates that there is a two thirds probability that the true maximum possible number of moneyers in type 3 lies within 35 of 192, i.e., between 157 and 227.

We can use the same process to obtain an estimate for Type 4 by considering the 56 moneyers from the 96 known in type 3 who are among the 108 moneyers known for type 4. This yields an estimate of 96×108/56 = 185 for the highest possible number of moneyers active in type 4. But we could get another estimate by considering the 55 moneyers from 136 known for type 2 who are found among the 108 known for type 4. This number, 55, is found in the row labelled type 2 and column labelled i=1 in Table 1. In this case the estimate of the highest possible number of type 4 moneyers is 136×108/55, which is 267. This is higher than the previous estimate of 185, but this is to be expected because a number of additional moneyers would have retired during type 3 leading to a lower retention rate. After the numbers 267 and 185 are multiplied by their respective retention rates (0.69 and 0.83, see below) they both give similar numbers (184, 156 respectively) as estimates of the total number of moneyers of type 4. At this stage we do not know what the retention rates are, but the ratio of 185 to 267 does give us an estimate of the retention rate of type 3. Given the large uncertainties in the values of 185 and 267 this estimate is not very accurate, but the example shows that the information we are looking for can be found in the numbers given in Table 1. A more accurate method of finding the retention rate is described below.

As a symbol for an estimate of the maximum possible number of moneyers, e.g., 192 for type 3 and 185 for type 4, it is convenient to use a lower-case n, with suffixes t,i to denote that it is based on the survival of moneyers known in type t after a gap of i intermediate types; in other words the estimate $n_{t,i}$ is the upper limit on the number of moneyers active in type $t+i+1$ based on the number surviving from type t. The estimates of $n_{t,i}$ from all the recorded combinations of starting and finishing types are given in Table 2. The number shown on the row labelled type 3 in the column labelled $n_{t,0}$ is an estimate of the upper limit of the number of moneyers in type 3 based on the known moneyers in types 2 and 3. The number $n_{2,1}$ in the next column is an estimate of the upper limit of the number of moneyers in type 4 based on the known moneyers of type 2 and 4. The estimates for type 4 are thus found in column 3 opposite type 4, in column 4 opposite type 3 and in column 5 opposite type 2 along an upward trending diagonal line.

The values of $n_{t,0}$ are plotted as elongated crosses in Fig. 1, where a pattern can be seen: the values of $n_{t,0}$ have well defined periods during which they tend to be constant. Five such periods can be identified, corresponding to Types 2–7 (period A), 8–12 (period B), 14–23 (period C), 24–27 (period D) and 28 (period E) with Types 1 and 13 being transitional. Period E, (type 28) represents the end of the *renovatio monetae* following the purge of the moneyers in 1124. The estimated uncertainties in the values of $n_{t,0}$ are shown by the vertical lines in Fig. 1. It clear that the actual values of $n_{t,0}$ within a given period are much closer to each other than would be expected from the estimated uncertainties. The reasons for this are discussed in the Appendix.

TABLE 1. Input values of the number of known moneyers in type t, m_t, and the number of moneyers known in type t as well as in type $t+i+1$, $b_{t,i}$, from Allen[29]

Type		m_t	$b_{t,t+i+1}$ $i = 0$	1	2	3	4	5	6	7	8
0	Ha	149	48	86	57	64	63	37			
1	W1–1	94	61	43	33	36	22	22			
2	W1–2	136	68	55	56	32	24				
3	W1–3	96	56	48	29	24	30				
4	W1–4	108	71	44	35	47	23				
5	W1–5	127	55	41	61	41	40				
6	W1–6	82	39	50	35	31	23				
7	W1–7	89	61	40	34	23	11				
8	W1–8	178	73	76	54	21	23				
9	W2–1	109	73	44	18	19	17				
10	W2–2	154	81	33	32	24					
11	W2–3	134	43	39	35						
12	W2–4	72	33	28							
13	W2–5	68	36	25	19	16	12				
14	H1	52	24	13	13	5	7	8	5	15	9
15	H2	54	17	13	8	10	10	3	11	8	
16	H3	44	14	10	4	9	3	11	11		
17	H4	42	11	5	11	5	14	9			
18	H5	30	9	9	3	11	9				
19	H6	28	7	5	8	7	12				
20	H9	37	6	16	15	16	16				
21	H8	20	10	9	13	5	10				
22	H7	68	29	35	11	22	31				
23	H11	55	39	15	24	31	6				
24	H10	113	27	37	66	13	7				
25	H12	45	27	34	4	4					
26	H13	94	66	15	10						
27	H14	136	17	13							
28	H15	110	53								
29	S1	158	28								
30	S2	61	25								
31	S6	46	22								
32	S7	98									

Note: In each case the row index is the initial type, t, used in the calculation and the column index is the number of intervening types, i.

TABLE 2. Estimate of the maximum possible number of moneyers, $n_{t,i}$, calculated using equation A2

Type		$n_{t,i}$ $i = 0$	1	2	3	4	5	6	7	8
1	W1–1	292	236	251	251	300	330			
		(57)	(38)	(47)	(45)	(52)	(71)			
2	W1–2	210	210	308	332	350	380			
		(39)	(44)	(69)	(71)	(92)	(99)			
3	W1–3	192	267	308	349	504				
		(35)	(50)	(56)	(79)	(124)				
4	W1–4	185	254	271	356	570				
		(36)	(50)	(65)	(90)	(127)				
5	W1–5	193	201	275	409	512				
		(34)	(42)	(61)	(78)	(127)				
6	W1–6	189	276	371	338	489				
		(37)	(57)	(64)	(69)	(97)				
7	W1–7	187	292	255	407	478				
		(41)	(57)	(57)	(92)	(120)				
8	W1–8	260	243	403	519	583				
		(47)	(52)	(88)	(129)	(198)				

[29] Allen 2012b.

TABLE 2. *Continued.*

Type		$n_{t,i}$ $i = 0$	1	2	3	4	5	6	7	8
9	W2–1	266 (45)	361 (57)	442 (79)	610 (158)	526 (133)				
10	W2–2	230 (39)	332 (66)	436 (122)	390 (106)	333 (98)				
11	W2–3	255 (41)	336 (76)	327 (75)	334 (87)					
12	W2–4	224 (47)	234 (51)	199 (47)						
13	W2–5	148 (36)	134 (35)							
14	H1	98 (24)	147 (40)	157 (47)	179 (57)	170 (62)				
15	H2	117 (33)	176 (61)	168 (58)	312 (157)	208 (93)	241 (100)	208 (106)	236 (75)	318 (120)
16	H3	140 (44)	174 (60)	203 (85)	151 (59)	200 (76)	360 (228)	334 (118)	371 (149)	
17	H4	132 (45)	132 (52)	308 (171)	181 (73)	293 (187)	272 (97)	220 (80)		
18	H5	115 (44)	235 (120)	141 (53)	168 (88)	204 (68)	257 (100)			
19	H6	93 (40)	123 (51)	200 (129)	185 (69)	183 (74)				
20	H9	148 (67)	112 (60)	238 (100)	220 (98)	264 (94)				
21	H8	123 (61)	157 (51)	136 (45)	261 (82)	104 (35)				
22	H7	136 (55)	122 (52)	174 (64)	180 (94)	188 (75)				
23	H11	129 (33)	220 (50)	278 (99)	291 (77)	298 (70)				
24	H10	159 (37)	165 (54)	215 (57)	241 (58)	1,080 (444)				
25	H12	188 (49)	287 (62)	233 (41)	956 (294)	2,551 (1,014)				
26	H13	157 (41)	188 (44)	1,268 (656)	1,778 (938)					
27	H14	194 (35)	689 (203)	1,485 (508)						
28	H15	880 (241)	1,653 (498)							
29	S1	328 (61)								
30	S2	344 (83)								
31	S6	112 (31)								
32	S7	205 (57)								

Note: In each case the row index is the initial type, *t*, used in the calculation and the column index is the number of intervening types, *i*. The standard uncertainties calculated using equation A19 are shown in parentheses. Estimates of the maximum number of moneyers for the same type lie on a diagonal line tending upward towards the right.

Retention rates and type duration

Since the values of $n_{t,0}$ are relatively constant within each period, the number of active moneyers and the retention rate must have either increased or decreased in the same proportion between different types within the period, or more likely, both remained constant. In either case we can obtain a more accurate estimate by averaging the values of $n_{t,i}$ that we expect

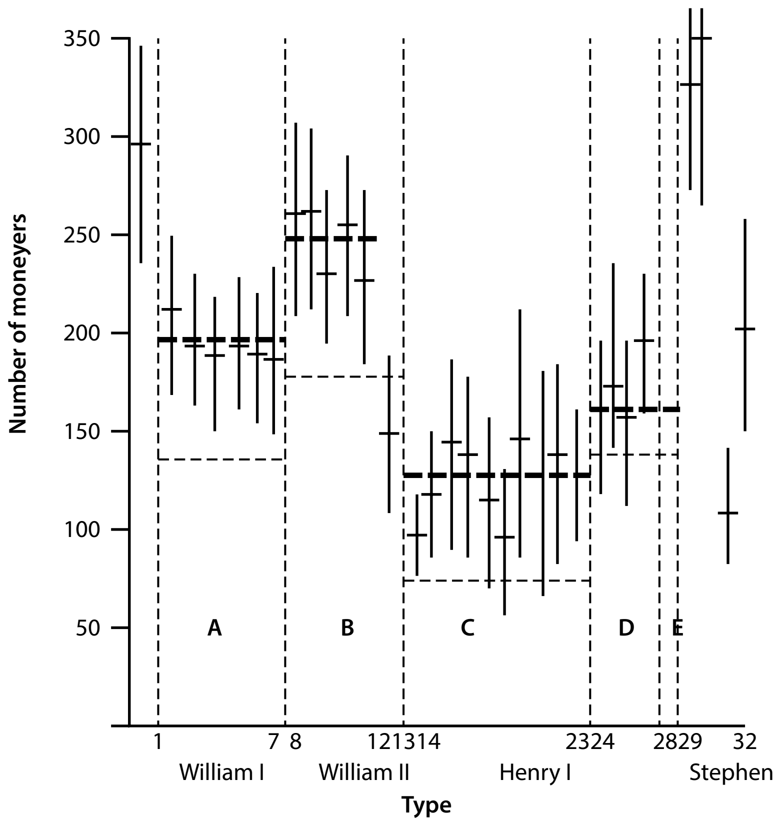

Fig. 1. Plot of the numbers of moneyers active in different types during the Norman period.
Note: The points (crosses) are the maximum possible number, $n_{t,0}$, calculated from equation A2, with the statistical uncertainty calculated using equation A19 shown by the vertical lines. In each of the four periods A, B, C and D the heavy broken line marks $n_{p,0}$, the average value of $n_{t,0}$ for the period, and the lower broken line is the corresponding value of the largest number of moneyers known in any type of the period. The number of moneyers active during a given type should lie between these limits.

represent the same number. These averages (with their standard uncertainties) are shown in Table 3. They are labelled $n_{p,i}$ where p identifies the period. A, B, C or D.

TABLE 3. Average values, $n_{p,i}$, for different values of i in each of the first four periods

p	t	$n_{p,i}$							
		$i = 0$	1	2	3	4	5	6	7
A	2–6	194	233	296	340	350			
		(3)	(16)	(12)	(8)				
B	8–12	247	318	427	564	583			
		(8)	(26)	(12)	(46)				
C	14–21	124	153	202	196	215	291	271	236
		(6)	(13)	(21)	(24)	(21)	(36)	(63)	
D	24–27	175	211	224	241				
		(10)	(38)	(9)					

Note: The types used in calculating the averages are shown in the column labelled t, types close to a transition are excluded. The number in parenthesis is the standard uncertainty in $n_{p,i}$ calculated using equation A18. No standard uncertainty is shown for the last item in each period as it is represented by a single value of $n_{t,i}$. Two thirds of the true values of $n_{p,i}$ are expected to lie within one standard uncertainty of the value shown.

The estimate of the total number of active moneyers during each of the periods is given by multiplying the numbers shown in Table 3 by the appropriate retention rate. The increasing trend in the estimates of $n_{p,i}$ with increasing i shown in Table 3 is a result of the decrease in the retention rate with the increase in the number of intervening types, *i*. Since the total number of active moneyers is assumed to be constant during a given period, the values of $n_{p,i}$ increase in the same proportion as the retention rate decreases. This allows us to obtain an estimate of the retention rate between any two types within any of the four periods. For example, in period A the retention rate between one type and that immediately following is given by 194/233 = 0.83, and the retention rate between two types with one intermediate type is 194/296 = 0.66, values that are shown in Table 4. As expected, these retention rates, plotted in Fig. 2, show a downward trend, which is steepest for period B (x), less steep for periods A (+) and C (o), and least steep for period D (*).

TABLE 4. Retention rates for different numbers of intervening types, *i*, calculated using equation A13 for each of the four periods

Period	*i* = 0	1	2	3	4	5	6
A	0.83	0.66	0.57	0.55			
	(5)	(4)	(2)				
B	0.78	0.58	0.44	0.42			
	(7)	(4)	(4)				
C	0.81	0.61	0.63	0.58	0.43	0.46	0.53
	(8)	(8)	(10)	(8)	(7)	(12)	
D	0.83	0.78	0.72				
	(16)	(9)					

Note: The numbers shown in parentheses are the estimated standard uncertainties in the second decimal place. They are derived from the standard uncertainties of the corresponding value of $n_{p,i+1}$ given in Table 3.

Also shown in Fig. 2 are three lines showing retention rates that would be expected if the annual retention rate is 0.91, i.e., on average 9 per cent of the moneyers retire each year. The upper curve is the one that would be expected if the duration of each type were one year, the next is if it were two years, and the lowest curve is if it were three years. The points for period A (+), which should be the most accurate, lie close to the two year curve except for the last point which lies above the curve. For larger values of *i* one would expect the retention rates to

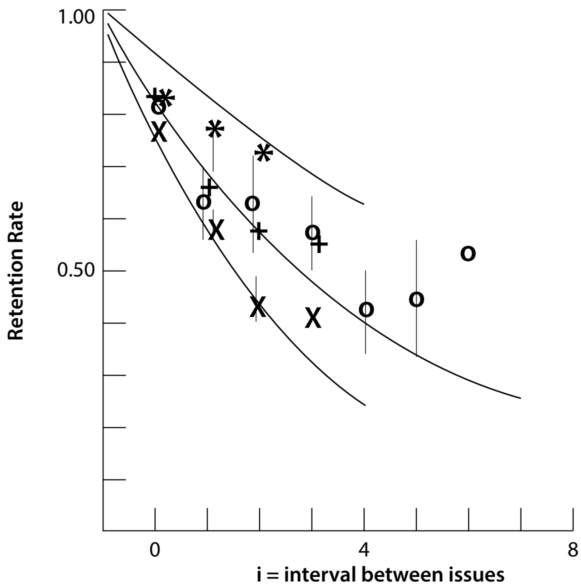

Fig. 2. The retention rates for periods A (+), B (x), C (o) and D (*) as a function of *i* calculated using the values of $n_{p,i+1}$ in equation A13.
Note: The three lines, reading from the top, show the expected retention rates calculated using equation A10 for a type duration of one, two or three years respectively for an annual retention rate, *Z*, of 0.91. The vertical lines indicate the standard uncertainty.

lie above the predicted curve since some of the newly appointed moneyers are likely to have the same names as moneyers who have recently retired.[30]

The first three points for period B (x) lie along the line expected for a duration of three years. The first three points for period C (o) where the uncertainty is larger, are similar to those for period A, but show more scatter, while period D (*) is represented by only three points which are subject to a relatively large uncertainty. It is difficult to assign an unambiguous duration to this period, but it appears to be not more than two years, possibly only one.

Determination of the number of moneyers

The duration of each type suggested by Fig. 2 during each of the four periods is given in column 3 of Table 5, and the retention rate, both that observed in Fig. 2 and that calculated from an annual rate of 0.91, are given in columns 4 and 5 respectively. These numbers are used with $n_{p,0}$ (column 2) to calculate the total number of moneyers active during each type of the period (column 6). Not all of these moneyers were active at the same time since some will have retired and been replaced during the course of the type. One might expect that the number of moneyers who were active at the beginning of a new type would be a measure of the number in office at any given time. This is given by the total number for the type multiplied by the retention rate and is shown in column 7. It is noteworthy that, except during period C, the number of moneyers active at any given time between 1066 and 1124 remained constant at around 140.

TABLE 5. Number of moneyers and retention rates by period

Period	$n_{p,0}$	*Duration (years)*	*Retention observed*	*Retention calculated*	*Moneyers total*	*Moneyers active*
A	194 (3)	2	0.83	0.83	161 (3)	138 (2)
B	247 (8)	3	0.76	0.76	188 (6)	143 (5)
C	125 (6)	2	0.83	0.83	104 (6)	86 (5)
D	175 (10)	1?	0.85–0.89	0.91	159*	145*

Notes: The numbers in parentheses are the standard uncertainties. The true value lies within two standard uncertainties at the 95 per cent confidence level.
* The uncertainties in the number of moneyers in period D depend on the uncertainty in the duration which is assumed to be one year (see the discussion in the text).

Proposed chronology of the types during the first three Norman reigns

The above analysis depends on a number of assumptions that, while plausible, may be questioned. Any interpretation therefore needs to be aware of these assumptions. The first is the assumption, strongly suggested by the results shown in Fig. 1, that the number of moneyers and duration of each type did not change within each of the four periods labelled A, B, C and D. While this assumption seems to be sound for the first three periods, it may not be entirely valid in period D, which would benefit from further study. A second assumption is that the annual retention rate is constant throughout the Norman period, except obviously during transitions between periods in which the number of moneyers was reduced. A third assumption is, that within each period, the type duration was a whole number of years. Both these last two assumptions are supported by the way the points follow the lines plotted in Fig. 2, and if valid, would suggest that the change in type always occurred at the same time of year, likely at a quarter day. The consistency of the values of $n_{t,i}$ during each period makes these assumptions plausible, at least for Periods A, B and C. Apart from the more problematic Period D

[30] The amount by which the points lie above the line suggest that after about fifteen years around half of the moneyers bearing a previously used name are more recent appointees, some possibly being sons of the original moneyer.

discussed in more detail below, nothing in the study is inconsistent with these assumptions. There is no evidence for an interruption of the pattern on the accessions of William II or Henry I.

The values of the type durations inferred for the four different periods (Table 5) are consistent with the available, if rather meagre, external evidence. The total duration of the William coins derived from six types of two years' duration in period A (including type 1) and seven types of three years' duration in period B is 33 years compared with the 34 year length of the two reigns; the total duration of the Henry coins derived from ten types of two years' duration in period C and four types, assumed here to be of one year's duration, in period D is 24 years, equal to the 24 years between the accession of Henry I in 1100 and the purge of the moneyers in 1124. The overall agreement of the proposed type durations with the regnal years supports the validity of the present study since the only variable that affects the absolute duration of the different types is the annual retention rate which cannot be very different from 0.91. The retention rate of 0.83 expected for a two year type-duration agrees well with the rates observed for periods A and C shown in Fig. 2. This is also close to the rate of 0.85 proposed by Blackburn for Henry's reign.[31] The retention rate of 0.76 expected for a three year duration agrees with the retention rate observed for period B as well as being consistent with the estimated retention rate of 0.72 that one would expect if the large Beauworth hoard included coins from all the moneyers active in type 8 (W1–8). Only the retention rate for Period D is ambiguous. It is certainly no less than the 0.83 expected for two years, and is likely to be greater. Except as noted below, a duration of one year is assumed in this analysis. It is consistent with the statistical results and best matches the regnal dates.

The chronology that results from the different type durations shown in Table 5 is given in Table 6. The duration of type 1 (W1–1) is assumed to be three years in order to match the regnal years, an assumption consistent with the large value of $n_{1,0}$. The changes that took place at the time of the conquest need to be more carefully analysed taking into account the two changes of monarch occurring during 1066, presumably combined with a change in the administrative arrangements. William was not crowned until Christmas of 1066 and the assumption of a three year duration is consistent with the York hoards being related to the Northern Rebellion, with type 2 (W1–2) being introduced around 1069. The proposed chronology would date the beginning of the snicking of the coins in type 19 (H6) to around 1110, some two years later than Eadmer's evidence that snicking began at some time around 1108. The proposed chronology dates the PAXS coins, type 8 (W1–8), to 1082–5, which suggests that it was not the first type of William II, this would have been type 10 (W2–2) rather than type 9 (W2–1) as in Brooke's numbering of the types. It is interesting that the Tamworth hoard contained coins of types 8, 9 and 10 (W1–8, W2–1 and W2–2), suggesting that type 9 was terminated prematurely after just one

TABLE 6. Proposed chronology

Type		*Period*	*Dates*	*Type*		*Period*	*Dates*
1	W1–1	A	1066–*c*.1069	15	H2	C	*c*.1102–*c*.1104
2	W1–2	A	*c*.1069–*c*.1071	16	H3	C	*c*.1104–*c*.1106
3	W1–3	A	*c*.1071–*c*.1073	17	H4	C	*c*.1106–*c*.1108
4	W1–4	A	*c*.1073–*c*.1075	18	H5	C	*c*.1108–*c*.1110
5	W1–5	A	*c*.1075–*c*.1077	19	H6	C	*c*.1110–*c*.1112
6	W1–6	A	*c*.1077–*c*.1079	20	H9	C	*c*.1112–*c*.1114
7	W1–7	B	*c*.1079–*c*.1082	21	H8	C	*c*.1114–*c*.1116
8	W1–8	B	*c*.1082–*c*.1085	22	H7	C	*c*.1116–*c*.1118
9	W2–1	B	*c*.1085–*c*.1088	23	H11	C	*c*.1118–*c*.1120
10	W2–2	B	*c*.1088–*c*.1091	24	H10	D	*c*.1120–*c*.1121
11	W2–3	B	*c*.1091–*c*.1094	25	H12	D	*c*.1121–*c*.1122
12	W2–4	B	*c*.1094–*c*.1097	26	H13	D	*c*.1122–*c*.1123
13	W2–5	B/C	*c*.1097–1100	27	H14	D	*c*.1123–1124
14	H1	C	1100–*c*.1102	28	H15	E	1125–1135

[31] Blackburn 1990, 66.

year, before all the coins of type 8 had been fully recoined.[32] If the accession of William II triggered a new type in 1087, the last type of William I, type 9, according the proposed chronology, would have been truncated. However, the present study shows no evidence for a short duration of type 9 and there are other explanations for why the Tamworth hoard might contain three different types.

The proposed chronology is subject to refinement, particularly in the years between 1120 and 1124. Like the other proposed chronologies reviewed by Allen,[33] the one presented here should not be considered absolute; shifts of a year or so either way are certainly possible.

Implications for Norman numismatics

The Norman period began with the conquest of 1066, a year that would profit from a more detailed analysis than is given here, reaching back into the reign of Harold II and last types of Edward the Confessor. In the present chronology type 1 is assumed to have lasted until 1069.

Types 2 to 6 (W1–2 to W1–6) clearly belong to period A, each with a duration of two years. The five types of period A (excluding type 1) therefore lasted for around ten years ending around 1079. A total of between 155 and 167 moneyers is estimated to have produced coins during each of these types, and the retention rate of 0.83 suggests that between 134 and 142 of these moneyers were active at any given time. These ranges represent the 95% confidence limits.

Fig. 1 shows that $n_{t,0}$ changed between types 7 and 8 (W1–7 and W1–8), which from equation A2, indicates that the retention rate changed during type 7, implying that the transitions between the two and three year duration occurred between types 6 and 7. This change may have coincided with a small increase in the weight of the penny.[34] In period B the number of moneyers active at a given time remained at around 140, but because of the longer type duration, the number of moneyers signing the coins in a given type increased from around 161 to 188. The 140 or so moneyers who were active at the end of type 6 were all in office for type 7 but now they issued coins of type 7 for three years rather than two.

The transition from period B to C is more problematic. Type 13 with $n_{t,0}$ equal to 148 appears to be transitional between period B ($n_{B,0}$ = 247) and period C ($n_{C,0}$ = 125). Although the individual values of $n_{t,0}$ are subject to larger statistical uncertainties than the values $n_{p,0}$ averaged over the whole period, they can still be used to estimate the total number of moneyers in the different types providing the retention rates are known, albeit with less accuracy. Two hypotheses are tested. In hypothesis A the number of moneyers active at a given time is reduced from 142 to 86 after type 12 but the duration remains at three years until the end of type 13. Hypothesis B is the reverse: the duration of type 13 is reduced to two years, but the reduction in the number of moneyers does not occur until after type 13. The expected retention for each type can be calculated for the two hypotheses. During period B where the type duration is three years the retention rate is 0.76, while during period C where the type duration is two years the retention rate is 0.83. However the retention rate for the type preceding the reduction in the number of moneyers is 0.46, given by dividing 86, i.e., the number of moneyers who were active at the beginning of following type (period C), by the total number moneyers who were active during the type immediately prior to the reduction. For each of the types at the time of the transition, Table 7 shows the values of $n_{t,0}$, the assumed retention rates for each hypothesis, and the corresponding resulting total number of moneyers estimated to have signed coins during that type. The numbers in parentheses are the number of moneyers that we would expect according to each of the hypotheses calculated using the averaged parameters for the periods B and C. It is clear that hypothesis A provides a better fit and also agrees better with the known regnal dates, but the large statistical uncertainties in the individual values of $n_{t,0}$ mean that we cannot completely rule out hypothesis B. In constructing the

[32] Stewart 1992, 129–32; Brown and Clarke 2008.
[33] Allen forthcoming, XXX.
[34] Metcalf 1988.

chronology in Table 6, I have assumed that hypothesis A is correct: that the number of moneyers active at any given time was reduced by about 54 in 1097, but the type duration was not shortened from three to two years until the accession of Henry I in 1100. It is possible that the decision to reduce the type duration was not made until 1102.

TABLE 7. Analysis of the transition between periods B and C testing the two hypotheses, A and B, described in the text

t	*Duration (years)*	$n_{t,0}$	*Retention A*	*Retention B*	*Total moneyers A*	*Total moneyers B*
11	3	255	0.76	0.76	175 (188)	175 (188)
12	3	224	0.46	0.76	193 (188)	193 (188)
13	3 or 2	148	0.76	0.46	103 (114)	170 (188)
14	2	98	0.83	0.83	112 (104)	68 (104)
15	2	117	0.83	0.83	81 (104)	81 (104)

Note that the calculation of the total number of moneyers is based on the value of $n_{t,0}$ and the retention rate for the previous type.

It is surprising that no one has previously noted the dramatic reduction in the number of active moneyers at the end of William II's reign, though Blackburn hinted at it.[35] The drop in the number of known moneyers at this time has tacitly been assumed to be related to the equally drastic drop in the number of surviving coins, but as shown in the Appendix, the results of the present study do not depend on the number of surviving coins and they are thus able to reveal the extent of the reduction in both the number of active moneyers and the type duration. The reduction by 39 per cent in the number of active moneyers would suggest that many mints were closed during period C. Given the low survival rate, not many mint names from this period are known and a study of possible mint closures is beyond the scope of his paper.

The present analysis of period D shows that the number of moneyers active at any given time was restored from 86 to 140 but otherwise it presents an ambiguous picture of the final years before the purge of 1124–25. Not only does the length of this period, covering only four or five types, result in larger statistical uncertainties in the estimate of the retention rate, but the results of the statistical analysis are themselves inconsistent. It is clear both from the regnal dates and the statistically determined duration times that the average type duration cannot have been much more than a year. This barely leaves enough time to complete one recoinage before starting the next.[36] It hints at an increasingly tight regulation of the moneyers, and a possible breakdown in a regular type duration. One cannot rule out the possibility that each recoinage was ordered as a response to a new monetary crisis. The fiscal problems that seem to have troubled Henry I were probably coming to a head during these years, since it is unlikely he would have ordered such a draconian purge in 1124–25 unless other less drastic attempts had failed to address the problems that the purge was intended to solve. One such remedy may have been responsible for the larger flans and possible increase in weight of coins of type 26 (H13).[37] The significantly large number of single finds of type 24 (H10) relative to most other types in periods C and D might seem to imply that this type had a longer duration than any other type before the purge, but the present study only measures the length of time a type was in production, while the number of single finds could be related to the level of output in the

[35] Blackburn 1990, 61.
[36] Brown 1997.
[37] Stewart 1989.

type as well as its duration.[38] If the *renovatio* system was indeed on the point of collapse in the years leading up to the purge, the introduction of a new type might not necessarily have been accompanied by the complete withdrawal of the previous type.

It is possible that signs of the impending collapse of the *renovatio* system triggered a breakdown in the regular type durations as early as type 23 (H11). The increase in the value of $n_{t,0}$ occurs between types 23 and 24, suggesting that period D begins with type 23. If this were the case, the average duration of a type during period D would be somewhat longer than one year. If the change started with type 24, as I have assumed in the chronology in Table 7, the average type duration in period D would be exactly one year. Blackburn pointed out a significant stylistic change introduced with type 24, which might support this assumption.[39] The present analysis raises as many questions as it answers for this troubled period, but it provides some useful constraints on possible explanations of what happened between 1118 and 1124.

The collapse of the *renovatio monetae* was complete with the purge that occurred between Christmas 1124 and Twelfth Night 1125. This event is readily seen in the statistics presented in Tables 1 and 2, though as period E that followed the purge contained only one type 28 (H15), we cannot make use of averaging to improve the accuracy of the analysis. The high value of $n_{28,0}$ (880) clearly indicates a retention rate of the order of 0.1. The number of known moneyers using the same mint signature in both types 27 and 28 is 17 ($b_{27,0}$ in Table 1). The assumption is that these are the moneyers who escaped the purge, but they may have been mutilated and reappointed later, or two different people may have had the same name and mint signature. Allen summarizes the evidence for a small number (3) who are known to have avoided mutilation by paying a fine,[40] and Stewart argues that some moneyers were apparently transferred to other mints rather than dropped, though these would be seen as different people in the results presented here, since the moneyers are identified by both their personal and their mint name.[41] If one assumes that the annual retention rate of 0.91 continued to apply during the years following the purge, only 39 per cent of the total number of moneyers in type 28 would have been active at any given time. If the total of 110 known moneyers in type 28 is nearly complete, then the number of moneyers active at any given time during type 28 would be at least 43, which would mean that nearly 97 of the 140 active before Christmas 1124 would have been removed from office. The striking similarity between this and the number of 94 mutilated moneyers reported in the Margam Annals is probably no more than a coincidence. Blackburn suggested that 80–85 moneyers were purged,[42] and Allen has estimated 58–78.[43] Whatever the true number, the reduction in the number of moneyers was accompanied by closure of more than half of the mints in the country.

The picture of Norman coinage that emerges is one in which William I adopted the existing *renovatio* system of mints and moneyers active at the time of the conquest. The study suggests that he placed the recoinages on a regular two year cycle. He later apparently reduced the frequency of the recoinages so that each continued for three years, a scheme that was continued by William II until, at the end of his reign, he reduced the number of moneyers and presumably closed some of the mints, while Henry I seems to have restored the two year cycle. Rising problems during the early years of the twelfth century, whether caused by the money supply or the general economic conditions, led to increasing concern over the state of the coinage. Various remedies such as snicking were tried, but were unable to prevent the gradual collapse of the *renovatio* system during the years leading up to the final purge of 1124–25.

38 Blackburn 1990, 54.
39 Blackburn 1990, 58–9.
40 Allen 2009, 86.
41 Stewart 1989.
42 Blackburn 1990, 66–7.
43 Allen 2009; 2012a, 27.

APPENDIX

Statistical Analysis

This appendix presents the mathematical details of the analysis and a discussion of its accuracy.

1. Deriving the basic equations

Let M_t be the total number of moneyers who signed coins at one time or another in type t, where t is the type number running from 1 (Type 1 of William I) to 28 (Type 15 of Henry I). Let m_t be the number of moneyers whose coins are listed by Allen as being known for this type.[44] Clearly m_t can be no larger than M_t. Let $B_{t,i}$ be the total number of moneyers, known and unknown, who were active in both type t and type $t+i+1$, where $i = 0$, 1, 2, 3 etc., being is the number of types intervening between the two types in which the $B_{t,i}$ moneyers were active. Not all of these $B_{t,i}$ moneyers are represent among the surviving coins. Some may be represented in only type t, others only in type $t+i+1$, and others not represented at all. Let $b_{t,i}$ be the number of moneyers listed by Allen as known to have struck coins in both type t and $t+i+1$. It is not necessary that these moneyers are known to be active in any of the i intervening types. Clearly all the $b_{t,i}$ moneyers also belong to the group of $B_{t,i}$ moneyers, and $b_{t,i}$ cannot be larger than $B_{t,i}$. The case where $i = 0$ is a special case where the second type, $t+1$, immediately follows the first type, and where one might assume that under normal circumstances all of the active moneyers continued in office. In this case $B_{t,0}$ is the number of moneyers who were active at the time when the type changed. One can expect some exceptions where, for example, the total number of active moneyers was reduced as is known to have happened in 1124–25.

There is one value of M_t and one value of m_t for each of the twenty eight types, but the values of $B_{t,i}$ and $b_{t,i}$ form matrices, such as that shown in Table 1, with rows labelled by t and columns by i. The size of these matrices depends on how many values of i are required. Values of i as high as 7 were included in some cases, but those between 0 and 4 were found to be the most useful. The twenty-eight observed values of m_t, together with the much larger number of observed values of $b_{t,i}$ shown in Table 1, provide an input of over a hundred independent numbers. This significant excess in the number of input observations over the number of derived numerical results is a major contributor to reducing the statistical uncertainties in this analysis as discussed below.

Let $X_{t,i}$ represent the retention rate, that is, the proportion of the M_t moneyers active in type t who were also active in type, $t+i+1$. $X_{t,0}$ is the special case, with i equal to 0, of the retention rate between type t and the following type $t+1$. For each type we would like to know the number of moneyers, M_t, and the retention rate, $X_{t,i}$, but the only information we have is m_t and $b_{t,i}$ From the above definitions it follows that $M_tX_{t,\mathrm{i}}$ moneyers who were active in type t remained active in the later type $t+i+1$. Of these we know the names of only the subset, $b_{t,i}$, of the $M_tX_{t,i}$ moneyers who were active in both types. We know the names of m_t moneyers of type t and we expect that $m_tX_{t,i}$ of them will have been active in both types. Not all of these will be among those whose names we also know in type $t+i+1$; only the proportion given by the survival rate m_{t+i+1}/M_{t+i+1} will be, the remainder will be among the unknown moneyers of type $t+i+1$. We therefore expect the $b_{t,i}$ moneyers known to be active in both types to be a fraction m_{t+i+1}/M_{t+i+1} of these $m_tX_{t,0}$ moneyers.

Therefore

$$b_{t,i} \sim m_t X_{t,0} \times m_{t+i+1}/M_{t+i+1} \qquad \text{A1}$$

where the symbol ~ indicates the expectation that the two sides of this equation would be the same if statistical fluctuations were absent.

Equation A1 can be rearranged to give

$$M_{t+i+1}/X_{t,i} \sim m_t m_{t+i+1}/b_{t,\mathrm{i}} = n_{t,i} \qquad \text{A2}$$

in which all the terms in lower case on the right hand side are known and appear in Table 1. Those in upper case on the left are the unknown targets of this study. Equation A2 serves to define $n_{t,i}$ whose values are given in Table 2 together with their standard uncertainties (in parentheses) calculated as described below.

Equation A2 can also be written as

$$M_{t+i+1} \sim X_{t,\mathrm{i}} n_{t,i} \qquad \text{A3}$$

$n_{t,i}$ can be calculated from equation A2 and if the retention rate, $X_{t,i}$ is known, M_{t+i+1} can be estimated. As $X_{t,i}$ cannot be greater than 1.0, it follows that $n_{t,i}$ represents an approximate upper limit to M_{t+i+1}.

Unfortunately, we know neither M_{t+i+1} (the total number of moneyers in type $t+i+1$) nor $X_{t,i}$ (the retention rate, i.e., the proportion of all the moneyers who struck coin of type t who continued in office in type $t+i+1$). All we have is their ratio. We can, however, place some limits on $X_{t,i}$ which allow us to suggest possible values. It cannot be greater than 1.0, as mentioned above, and it cannot be smaller than $m_{t+i+1}/n_{t,i}$ since there must be at least m_{t+i+1} moneyers because we know their names. We can therefore determine that

[44] Allen 2012b.

$m_{t+i+1}/n_{t,i} \lesssim X_{t,i} < 1$ A4

where $\lesssim$ means that the inequality is subject to statistical uncertainty.

This means that

$m_{t+i+1} < M_{t+i+1} \lesssim n_{t,i}$ A5

where both limits are known.

If $i = 0$ the second type follows immediately after the first type with no intervening types and we can write

$m_{t+1} < M_{t+1} \lesssim n_{t,0}$ A6

$X_{t,0}$ represents the proportion of all the moneyers active in type t who were still active at the beginning of the following type, $t+1$, and as such it provides an indication of the duration of type t, since the longer the duration of the type, the smaller the value of $X_{t,0}$. It is possible to make a rough estimate of $X_{t,0}$. In the absence of the wholesale removal of moneyers, such as occurred in 1124–25, we might assume a constant annual retention rate, Z, namely the proportion of the moneyers active at the beginning of any given year who were still active at the beginning of the following year. If this assumption is valid, the value of $X_{t,0}$, the retention rate between one type and the next, would depend only on the duration, Y years, of the type as given by equation A7.

$X_{t,0} = Z^Y$ A7

For example, if the annual retention rate were 0.9, half of the moneyers would still be active seven years after they were first appointed. In this case $X_{t,0}$ would have value of 0.90, 0.81 or 0.73, depending on whether the duration of type t was one, two or three years respectively. The assumption that Z remained constant is of course open to question since more moneyers may have been employed during the recoinage in the first year of the type, or more may have retired at a type change, but as long as Y was a whole number of years, as seems probable, it should be possible to distinguish between durations of say two or three years. It is unlikely that one could reliably measure fractions of a year.

While the value of Z is not likely to deviate too far from 0.9, a more precise value can be determined by exploring the retention of moneyers over a period of several types, that is, using the values of $b_{t,i}$ with non-zero values of i. In this way we can draw on a much larger body of independent information.

In this extended picture, $n_{t,i}$ represent an additional upper limit on the number of moneyers who signed coins in the final type, $t+i+1$ (Table 2). The values of $n_{t,i}$ that refer to the same final type, i.e., those values for which $t+i+1$ = constant, lie not along a row or column of Table 2, but on a diagonal running from the bottom left to the top right of the matrix with t becoming smaller as i increases.

We expect the retention rates, $X_{t,i}$, to become smaller as i increases, since the longer the time between the initial and the final type, the more moneyers will have retired. Therefore according to equation A3, the values of $n_{t,i}$ should increase in the same proportion that the retention rate diminishes since M_t is the same for all i. This trend can be seen along the diagonals in Table 2, though the statistical uncertainties result in a number of exceptions, particularly in Period C where the sample size is small.

Even a cursory glance at Table 2 shows that the values of $n_{t,0}$ typically remained essentially constant over a number of consecutive types, allowing the coins issued between 1066 and 1124 to be divided into four periods labelled A, B, C and D during each of which the values of $n_{t,0}$ are similar with well defined transitions between the periods. It is a reasonable assumption that all the estimates of $n_{t,i}$ that lie within the same period are estimates of the same quantity, in which case it is permissible to replace the individual values of $n_{t,i}$ by the averages, $n_{p,i}$, over all types in period p (= A, B, C or D) since the average will be a better estimate than the individual values.

Provided both types t and $t+i+1$ lie within the same period, the values of M_{t+i+1} are all assumed to be equal to the same quantity, M_p. Similarly all the values of $n_{t,i}$ with the same value of i should be the same and equal to $n_{p,i}$ which is defined by equation A8.

$n_{p,i}$ = average over t of $n_{t,i}$ for the period p. A8

Equation A3 then can be written as equation A9.

$X_{p,i} \sim M_p/n_{p,i}$ A9

Equation A9 gives the best estimate of the retention rate between two types with i intervening types all within the same period. The values of $n_{p,i}$ are shown in Table 3 for each of the four periods together with their standard uncertainties in parentheses. As expected, Table 3 shows that $n_{p,i}$ increases with i with some exceptions in Period C. In all cases these exceptions lie well within the standard uncertainties obtained from the estimated standard deviations of the respective values of $n_{t,i}$.

Since the retention rate between sequential types, $X_{p,0}$, is not expected to change during a given period, following equation A7 we would expect $X_{p,i}$ to be given by equation A10.

$X_{p,i} = X_{p,0}{}^{i+1}$ A10

Hence

$X_{p,i} = X_{p,i+1}/X_{p,0}$ A11

and since

$$X_{p,i+1} \sim M_p/n_{p,i+1} \text{ and } X_{p,0} \sim M_p/n_{p,0} \quad \text{A12}$$

by combining A11 and A12 and cancelling out M_p, we get

$$X_{p,i} \sim n_{p,0}/n_{p,i+1} \quad \text{A13}$$

As the values of $n_{p,0}$ and $n_{p,i+1}$ are known (Table 3), equation A13 gives a direct estimate of the retention rates between any two types that are separated by i intervening types within the same period. These estimates of $X_{p,i}$ are given in Table 4 and plotted in Fig. 2 together with their standard uncertainties. As discussed in the text, the observed values of $X_{p,i}$ lie closely along the lines expected for integral values of Y.

2. Limits of uncertainty

2.1. Dependence of the results on the survival rate

S_t is the survival rate defined by equation A14.

$$m_t = M_t S_t, \quad \text{A14}$$

Since $b_{t,i}$ is affected by the survival rates of both types t and t+i+1

$$b_{t,i} \sim B_{t,i} S_t S_{t+i+1} \quad \text{A15}$$

Substituting equations A14 and A15 into equation A2 gives

$$n_{t+i+1} \sim (M_t S_t)(M_{t+I+1} S_{t+i+1})/B_{t,i} S_t S_{t+i+1}) \quad \text{A16}$$

and since S_t and S_{t+i+1} cancel, this gives

$$M_t M_{t+i+1}/B_{t,i} \sim n_{t+i+1} = m_t m_{t+i+1}/b_{t,i}$$

which is the same as equation A2, indicating that equation A2 is independent of the survival rate within the limits of statistical uncertainty.

2.2. Estimates of the standard uncertainty

The *standard deviation* is a measure of the spread that would be expected in the observed values of a quantity if several independent observations of the quantity are available. For spreads that result from random variation in the selection of the observed objects, such as the selection of surviving coins from the totality of coins produced, it is normally assumed that any given observation has a 66% chance of lying within one standard deviation of the true value, or a 95% probability of lying within two standard deviations. The standard deviation can be estimated (*the estimated standard deviation, esd*) if a number, *n*, of independent observations of the same quantity, *x*, are available.

$$esd = \sqrt{(\sum(x-x_{average})^2/(n-1))} \quad \text{A17}$$

where the sum is over the *n* different values of *x*

When the *esd* is regarded as a measure of the accuracy of an individual value of *x* it is called the *standard uncertainty (su)*. Since the average value of *x* should be a more accurate estimate of the true value than any of the individual values of *x*, its standard uncertainty is given by equation A18

$$su = esd/\sqrt{n} \quad \text{A18}$$

In the present study, a random selection would require all moneyers to have produced the same number of coins, which is clearly not true. If it were true and if M_t were much larger than m_t, we would expect the standard deviation of m_t to be equal to $\sqrt{m_t}$,. In this case the standard uncertainty in n_{t+i+1} would be given by equation A19.

$$n_{t,i}(1/m_t + 1/m_{t+i+1} + 1/b_{t,i})^{1/2} \quad \text{A19}$$

In practice the standard uncertainty in $n_{t,i}$ is expected to be much smaller than this for a number of reasons. Firstly, equation A19 is valid only when the survival rate, *S*, is small. As *S* approaches 1.0 the standard deviation is reduced since the standard deviation must be zero for *S* = 1.0. In the present study, survival rates vary from 0.1 to close to 1.0.

Secondly, it is known that some moneyers produced significantly more coins than others, an effect that also reduces the standard deviation when *S* lies in the middle of its range since the addition of a newly found coin to the list of surviving coins will have a higher probability of being the work of a prolific moneyer whose name is already included in m_t than of being the work of a less productive moneyer whose name is not yet known. The value of m_t is therefore relatively insensitive to changes in the survival rate.

Thirdly, a reduction in the standard uncertainty is achieved by averaging over all the types that belong to the same period, if as assumed, the number of active moneyers and type durations did not change within each period. More than a hundred independent observations of n_t and $b_{t,i}$ are used to produce just nine numbers (Z, and M_p and Y for each of four periods) resulting in a significant improvement in the accuracy.

A better estimate of the standard uncertainty than that given by equation A19 is obtained from equation A18 using the estimated standard deviations of the individual values of $n_{t,i}$ within each period as given by equation A17 and shown in Table A1. As an example, for Period A the estimated standard deviation of values of $n_{t,0}$ is 8 compared to 37 expected from a random selection model using equation A19. This indicates that the true standard uncertainty is less than a quarter of what would be expected if the selection were random. The standard uncertainty (*su*) of the average, $n_{A,0}$, calculated using equation A18, is even smaller, just 3, meaning that the true value of $n_{A,0}$ lies between 188 and 200 at the 95% confidence level. The results for the different periods are summarized in Table A1.

Table A1. Estimates of the standard uncertainty of $n_{t,0}$ and $n_{p,0}$

p	*T*	$n_{p,0}$	*A19*	*esd* *A17*	*su* *A18*	*Ratio*
A	2–6	194	37	8	3	0.08
B	8–12	247	43	17	8	0.19
C	14–21	125	45	17	6	0.13
D	24–27	175	41	17	10	0.24

Col. 2 gives the types that were used in calculating the averages $n_{p,0}$.
Col. 3 gives the average $n_{p,0}$ for each period (taken from Table 3).
Col. 4 gives the average of the standard uncertainties shown in Table 2 and Fig. 1 calculated using equation A19. This represents an upper limit to the uncertainty.
Col. 5 gives the estimated standard deviation (*esd*) of the values of $n_{t,0}$ calculated from equation A17.
Col. 6 gives the standard uncertainty (*su*) in $n_{p,0}$ calculated using equation A18.
Col. 7 gives the ratio of col. 6 to col. 4, showing the improved accuracy that results from the factors discussed in the text.

BIBLIOGRAPHY

Allen, M., 2009. 'Henry I type 14', *BNJ* 79, 72–171.

Allen, M., 2012a. *Mints and Money in Medieval England* (Cambridge).

Allen, M., 2012b. 'Mints and moneyers of England and Wales 1066–1158', *BNJ* 82, 54–120.

Allen, M., 2014. 'Coinage and currency under William I and William II', in R. Naismith, M. Allen and E. Screen (eds), *Early Medieval Monetary History: Studies in Memory of Mark Blackburn* (Farnham), 85–112.

Andrew, W.J., 1901. 'A numismatic history of the reign of Henry I, 1100–1135', *NC* 4th series, 1, 1–515.

Archibald, M. M. 1984. 'Coins', in G. Zarnecki (ed.), *English Romanesque Art 1066–1200* (London), 320–41.

Blackburn, M. 1990. 'Coinage and currency under Henry I: a review', *Anglo-Norman Studies* 13, 49–81.

Brown, I.D., 1997. 'Active mints and the survival of Norman coins', *BNJ* 67, 1–10.

Brown, I.D. and Clarke, W.N., 2008. 'The duration of the late Saxon and Norman coinage periods', *NCirc*, 116, 298–9.

Dolley, M., 1966. *The Norman Conquest and the English Coinage* (London).

Metcalf, D.M., 1988. 'Notes on the "PAXS" type of William I', *Yorkshire Numismatist* 1. 13–26.

Rule, M. (ed.), 1884. *Eadmeri historia novorum in Anglia*, Rolls Series 81 (London).

Stewart, I., 1989. 'Type XV of Henry I', *SCMB* 795 (Nov. 1989), 259–64.

Stewart, I., 1992. 'Coins of William II from the Shillington hoard', *NC* 152, 111–32.

Thompson, J.D.A., 1956. *Inventory of British Coin Hoards, A.D. 600–1500*, RNS Special Publication 1 (London).